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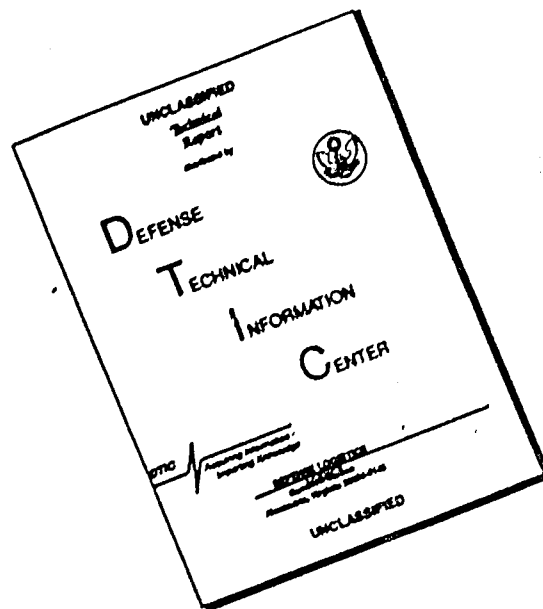
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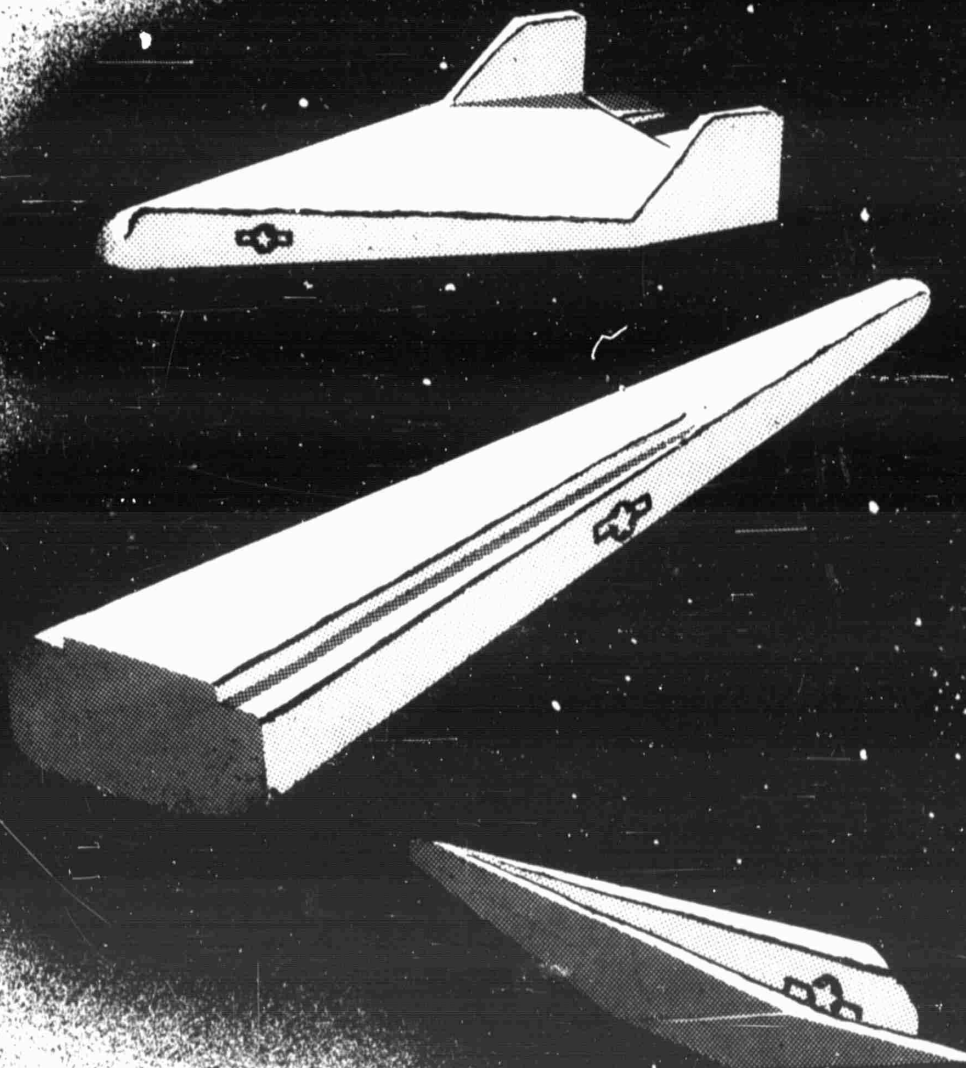
Flight Dynamics Laboratory

ASD

POWER SPECTRAL DENSITY TECHNIQUES APPLICABLE
FOR STRUCTURAL DESIGN CRITERIA AND ANALYSIS
STATE-OF-THE-ART REPORT

Leon J. Bowser   ASRMDS-11

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STATE-OF-THE-ART REPORT

LEON J. BOWSER

DESIGN CRITERIA SECTION
STRUCTURES BRANCH
FLIGHT DYNAMICS LABORATORY
DEPUTY FOR TECHNOLOGY

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AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

This report was prepared by Leon J. Bowser as an in-house effort under Project Number 1367, "Structural Design Criteria", Task Number 136702, "Aerospace Vehicle Structural Design Criteria". The work was accomplished under the direction of C. J. Schmid, project engineer.

This report covers work performed from February 1961 through December 1961.

ABSTRACT

A review and analysis of the status of the state-of-the-art regarding application of power spectral density techniques to structural design criteria is presented. The primary objective being to make available a convenient reference giving an organized, readable answer to the questions, "What has been done in applying Power Spectral Techniques?" and "What are some of the potentialities of this tool in helping to solve complex engineering and design problems?" In addition, the report includes a detailed account of some Aeronautical Systems Division experience in applying these techniques, and a brief introduction to spectral analysis techniques.

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PUBLICATION REVIEW

This report has been reviewed and is approved.

William C. Nielsen

WILLIAM C. NIELSEN
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I.   Introduction

This report contains a review and analysis of the state-of-the-art status regarding application of power spectral density techniques. This report does not presume to supersede any of the literature in the areas of generalized harmonic analysis, transform calculus, or random-process theory. Stated differently, this report is concerned solely with the application and exploitation of established methods and techniques, and not the definition or exposition of fundamental concepts of post-doctor-ate level mathematics. It is fully acknowledged that a philosophical analysis of these techniques is a legitimate study. However, in applica-tions, the abstract mathematical models serve as tools and different models can describe the same empirical situation. The manner in which mathematical theories are applied does not depend on the preconceived ideas; it is a purposeful technique depending on, and changing with experience.

A Moral

"To analyze time series effectively we must do the same as in any other area of statistical technique "Fear the Lord and Shame the Devil" by admitting that:

(1) The complexity of the situation we study is greater than the complexity of that description of it offered by our estimates.

(2) Balancing of one ill against another in choosing the way data is either to be gathered or to be initially analyzed always requires know-ledge of quantities which cannot be merely hypothesized and which, in many cases, we cannot usefully estimate from a single body of data, such as ratios of (detailed) variance components or extents of non-normality. Theoretical optimizations based upon specific values of such quantities may be useful guides. However, theoretical optimizations should be used only when the failure of past experiences and the present data to give precise values for these quantities is recognized and allowed for.

(3) There is no substitute for repetition as a basis of assessing stability of estimates and establishing confidence limits.

(4) Asymptotic theory must be a tool, and not a master.

The only difference is that one must be far more conscious of these acceptances in time series analysis than in most other statistical areas." Some concrete examples of these principles are briefly discussed in this section and expanded upon in subsequent sections and appendices.

---

*Dr John W. Tukey, Princeton U. Tech Rpt No. 91, March 1961.

In many structural design problems, the knowledge of the physical phenomena is not precise enough to justify exact predictions with respect to each individual observation. On the one hand, the loading is never exactly known, and on the other, the physical system may be so complicated that detailed calculations for the prediction of dynamic responses are difficult. (e.g. The random nature of the atmospheric turbulence as shown in the records is due to the influences of viscosity, pressure, density, temperature, humidity, and velocity distributions, varying in such a complicated manner that mechanistic predictions based on the hydro-dynamic equations are practically impossible.)

In such cases the physical data can be regarded as a set of statistical data of random processes. Logical statements of the forcing functions are therefore statistical--not mathematical but statistical. This is very important here because a mathematical statement usually makes an assertion which is true in each individual case, and if there is one case which violates the mathematical statement it also disproves the same statement. While a statistical statement ensures only that over the long run its truth will provide itself. For example, a typical statistical statement may be that the probability of obtaining heads in a single toss of a coin is 1/2. However, two consecutive tails surely is not sufficient to destroy faith in the statistical hypothesis.

If one recognizes then the uncertainty of predicting the forcing functions in a given problem, the problem immediately becomes statistical, and one must attempt to state the main features of the responses, as well as those of the forcing functions in terms of statistical averages and probability distributions. Another important example is structural fatigue under random repeated loading. Introductory material relevant to this problem is contained in Appendix II of this report.

II. Applications of PSD to the Gust Problem

An excellent summation of the state-of-the-art as of May 1957 is afforded by Dr Raymond L. Bisplinghoff, Professor of Aeronautical Engineering, MIT:

"It is not unreasonable to say that we have reached the place in the development of a new technique where it is no longer necessary to justify its validity or usefulness. It is true that there are still unanswered questions and inconsistencies, but the advantages to be derived from the application of power spectral analyses to the gust problem far outweigh these short comings. The principal problem to be faced now is how the process can be placed more efficiently in the hands of the structural designer. This is indeed the most difficult phase of the problem. In its present state, power spectrum analyses can and should serve as an adjunct to other methods prescribed by regulatory agencies, and one would expect that progressive engineers, especially those dealing with unconventional designs, would do so. Dynamic analyses by the power spectrum approach has not at the present writing reached the state where it can be applied with confidence to the art of airplane design as a completely rational and independent process. However, its present value to engineers in comparing the properties of one aircraft with those of another should not be underestimated. It has unquestionably reached a state where serious consideration should be given to incorporating it in some form in government design criteria.

Three places are suggested where effort is required to improve this situation. The first is continued effort to classify the atmosphere - especially under conditions of severe turbulence. Secondly, more explicit data on the range of turbulence scales to be encountered is needed. Finally there should be analyses of the time histories of older aircraft by means of which the power spectral dynamic analysis should be applied to older airplanes whose strength and service life are well known. In this way reference points can be established which reflect the past experience of design and service and from which the same methods of analysis may be used in a comparison and extrapolation process involving new designs. This is a large undertaking but perhaps if it were undertaken by the builders of the aircraft in question, it would be accomplished in reasonable time".

III. Limitations of Early PSD Analyses

Perhaps the most important omission in early analyses was the pitching degree of freedom and associated short period damping; especially for unconventional configurations involving swept, delta-type or tail-less arrangements. Also the effect of span-wise variation of gusts is important. The span-wise variation is often represented by $\frac{b, \text{ wing span}}{L, \text{ scale of turbulence}}$. For larger values of b/L, the average velocity along the span is progressively reduced particularly at the high frequencies and attenuation in wing lifts and in overall aircraft accelerations will result.

Not always so. Actually, the effect of taking the span wise variations of gust intensity into account is to reduce the mean squared bending moment when most of the mass is in the fuselage and to increase it when most of the mass is in the wing.

The magnitude of the effect for a flexible wing appears to be greater than for a rigid wing where the analysis is based on the first symmetrial free-free bending modes.

IV. Extensions of PSD Analyses and Comparison with Flight Results

PSD Analysis of the gust-load problem was extended by Diederich (Ref. 7) to include the effect of lateral variation of the instantaneous gust intensity on the aerodynamic forces. The forces obtained in this manner were used in dynamic analysis of rigid and flexible airplanes free to move vertically, in pitch, and in roll. The effect of the interaction of longitudinal, normal, and lateral gusts on the wing stresses were also considered.

The mean square values, correlation functions, and power spectra of some of the aerodynamic forces required in this type of analysis were calculated for one special correlation function of the atmospheric turbulence. It was shown that if the span is relatively large compared with the integral scale of turbulence, the mean square lift and root bending moment directly due to the gust are substantially reduced when the differences in instantaneous intensity of the turbulence along the span are taken into account. However, if the motions of the aircraft are taken into account the mean square root bending moment may be increased as a result of these differences.

Also, the mean square pitching moment was shown to be substantially increased if the tail length is relatively large compared with the scale of turbulence. Finally, the wing stress due to longitudinal, normal and lateral gusts were shown to be statistically independent under certain conditions. (Ref Fig. 1, 2, 3)

The effects of wing flexibility on the wing strains that result from gust encountered were examined by Shufflebarger (Fig. 4) on a power spectral basis. Both flight results and analytical results were considered for a four engine bomber aircraft, and correlations between the measured and analytical results were made. The effects of wing flexibility on the wing strains were measured in terms of amplification factors based on the ratio of the strain for the flexible condition to the strain for the "rigid" condition, and results were obtained for four spanwise stations of the wing. The effect of including the second symmetric bending mode in addition to the fundamental bending mode in the calculations was also evaluated. The study led to the following results.

1. Comparison of the measured strain spectra for the flexible and rigid conditions showed strain amplifications in the frequency range below 0.5 cps, especially at the outboard wing stations, that are attributed to the effect of factors such as spanwise gust variations and aircraft rolling motions rather than to flexibility effects.

2. The measured strain amplification factors due to wing flexibility at a station near the wing root were 1.09 on the basis of the rms strains derived from distributions of strain peaks. The amplification factors decreased with each successive outboard station and then increased slightly at the most outboard measuring station.

3. The calculated output spectra were in reasonable agreement with the measured spectra except at low frequencies, where the differences are attributed to extraneous factors. At the outboard stations it was necessary to include the effect of the second symmetric bending mode in the calculations in order to adequately estimate the effect of flexibility on the distribution of strain peaks.

4. Both calculations and measurements indicated that the second symmetric bending mode had a very small effect on the rms strains but that for strain amplification factors determined from distributions of strain peaks, the second symmetric bending mode had a pronounced effect at the outboard wing stations.

5. Although uncertainties in the calculations may arise from a number of factors; these results indicated that calculations based on PSD may provide reliable estimates of the magnitude and character of wing flexibility effects on wing strains in gusts for unswept-wing aircraft. Also, a flight investigation was made by Rhyne on a large swept-wing bomber airplane in rough air at 35,000 and 5,000 feet to determine the effects of wing flexibility on wing bending and shear strains. (Ref. Fig. 5) In order to evaluate the overall magnitude of the aeroelastic effects on the strains and their variation with spanwise location, amplification factors defining the ratio of the strains in rough air to the strains expected for a "rigid" and "quasi-rigid" aircraft were determined. The results obtained indicated that aeroelastic effects are rather large, particularly at the out-board stations. The effects of dynamic aeroelasticity appear to increase the strains from 0 to 170 percent depending upon the spanwise station. On the other hand, the relieving effects of static aeroelasticity appear to reduce the strain amplification in rough air by a significant amount. The response of swept-wing aircraft in rough air involves a number of complications not present in the case of unswept-wing aircraft due-principally to the increased importance of torsion for unswept-wing aircraft; the torsion in turn results in significant effects on both the aircraft dynamics and stability; in addition, the aircraft vibratory modes may no longer be approximated by simple beam bending theory but may require consideration of coupled bending-torsion modes. Few experimental data exist on the character and magnitude of these problems.

Results at 35,000 feet were compared with previous results at 5,000 feet.and comparison showed wing trains, on the average, about 20% larger at the higher altitude. Representative values of the amplification factors varied about 1.3 at the root equations to about 2.5 at the midspan stations.

The analysis of the strain responses of a large swept-wing airplane in rough air by Coleman, Press and Meadows by means of experimentally determined frequency response functions indicated that the wing-bending and shear strain responses at the various stations are amplified by rather large amounts because of the responses of the structure.

The amount of amplifications in bending strains was about 10 to 20 percent at the root stations but increased to values in excess of 100% in some cases at the midspan stations. The shear strains showed a similar pattern across the airplane span but also indicated larger variations between front and rear span stations. The large variations in strain responses across the airplane span indicated that strain distributions in gusts are very different under rough air loading conditions than under the usual maneuver loadings and warrant detailed and separate consideration in design. In general the predominant source of strain amplification was associated with the excitation of the fundamental wing-bending mode. However at the outboard stations and particularly in the case of the shear strains, significant contributions to the strains arise from the higher symmetrical and antisymmetrical vibration modes. Thus, the effects of these higher modes on the strains may also have to be considered in stress calculations depending on the degree of accuracy required. A detailed analysis of the reliability of frequency-response function estimates obtained by random-process techniques, particularly as affected by extraneous noise, was given. The effects of such noises in giving rise to systematic errors or distortions and random sampling errors were explored and results of general applicability obtained. These results were also applied to the present test data in order to establish their reliability and to derive adjustments for the distortions. The important result obtained is the indication that with appropriate precautions flight tests in rough air of few minutes duration may be used to obtain reliable estimates of airplane frequency-response functions.

The determination of aircraft frequency-response functions for responses to atmospheric turbulence from measurement in continuous rough air involved a relatively new application of random-process techniques. A general analysis of the reliability of such frequency response estimates was presented and methods of estimating the distortion and sampling errors were developed and applied.

V. Evaluation of Methods of Calculating Bending Moment Response of a Flexible Vehicle

·Sure to arise as a practical question when reviewing and comparing analytical results is: "Which results are better?"

Several approximate procedures for calculating the bending moment response of flexible aircraft to continuous isotropic turbulence were presented and evaluated by Bennett (Reference 2). These approximate procedures were applied to a simplified aircraft which consisted of a uniform beam with a concentrated fuselage mass at the center. The conclusions drawn from this study were:

1. The force-summation method based on one natural bending mode gives very good results compared with the exact solution.

· 2. The matrix method based on five stations across the semi-span gives very good results compared with the exact solution.

3. The mode-displacement method based on a natural mode yields inaccurate results; however this relatively simple method can be useful in trend studies involving variations in wing flexibility.

4. The force-summation method, based on an approximate parabolic mode and the Rayleigh frequency, loses little accuracy in mean-square results. However, the approximated natural frequency causes a shift in the fundamental mode resonant peak; consequently, higher statistical moments will be in appreciable error.

5. The force-summation method based on an approximate parabolic mode loses little accuracy if the natural frequency is known. The inclusion of spanwise variations of turbulence results in a decreased response if most of the aircraft mass is in the fuselage and in an increased response if most of the mass is in the wing. Also the exclusion of spanwise variations of turbulence in trend studies such as trend of amplification factor with mass ratio, may lead to erroneous conclusions.

VI. Safety and Safety Factors for Airframes

It is generally accepted that gusts are the dominating cause of fatigue with respect to the wings of transport airplanes, while the dominant fatigue damage of wings of fighter or trainer planes is due to maneuver loads.

The purpose of the application of statistical methods to problems of maximum loads and of load spectra is to obtain by extrapolation estimates of the probability of encountering the extremely high design load factors of very low expected frequency of occurrence as well as to obtain estimates of the rate of accumulation, under random loading associated with certain load spectra or fatigue or creep-damage. It should be obvious that, because of the necessity of such extrapolation, the selection of a probability function involves more than curve fitting. Unless the selected probability function is germane to the problem and adequately represents the inherent statistical variability of the phenomenon which results from certain basic assumptions concerning its origin, extrapolation towards the extremes (tails of the function) will result in erroneous predictions within this range of variation, which is just the relevant design range. It appears that the probability distribution of maximum values obtained from successive samples of records can be used for an effective estimate of the required extremes for design. The statistical theory of the distribution of extreme values indicates that for initial distributions of the most common type which approach zero exponentially, a limiting form exists for the distribution of the maximum values of large samples. The use of this asymptotic form should therefore increase the reliability of the estimate in comparison to direct extrapolation from the observed initial distribution.

The character of the probability function of extreme gust-intensities, with the aid of which the ultimate load factor can be estimated, is thus completely determined by the extremal nature of the phenomenon. It is independent of the nature of the underlying distribution of gusts.

If the underlying distribution of gusts is required it can either be obtained by statistical interpretation of actual gust records or by a theoretical study of the effects on the airplane of atmospheric turbulence and of buffeting, and of the response of the airplane to these random disturbances by means of generalized harmonic analysis and power spectra techniques.

The design gust spectrum should extend from the limit load downward. However, the small probability of actual occurrence, during the operational life, of loads in the vicinity of the limit load makes this load region rather insignificant with respect to damage accumulation over this life. The specific shape of the spectrum is therefore less significant in the limit load region than in the region of higher frequencies; thus, discrepancies between actual load distribution and the assumed form of the spectrum can be tolerated in the vicinity of the limit load. The

conclusion appears, therefore, to be justified that the initial frequency function of gusts can be fairly well represented by a simple exponential probability function over the entire range. The same can also be done for maneuver loads for fighters.

VII Measurement and Assessment of Repeated Loads

Further light was thrown on the principal problem--placing PSD techniques in the hands of the structural designer--by Donely (Ref. 6). He outlined the general principles vital to the measurement and assessment of repeated loads as follows:

1. Extremal loads that compromise the load history of an airplane should be treated according to load source. Indications are that there is no unique strain relation applicable to all points in the structure or all load sources.

2. The relative importance of the different load sources will vary according to mission and usage of the airplane. (Fig. 7).

3. It is mandatory to subdivide the load histories further according to flight condition or a source providing more homogeneous information. Loads and their sources:

Load	Source
Gust Loads	cloud and wind-shear turbulence, other meteorological factors
	flight conditions - climb, cruise, and descent. In the case of climb and descent although the gust experience should be the same, the difference in flight speed has a significant influence on the load distributions
Ground Loads	air-to-ground cycle, landing impact, taxi loads

4. On the assumption that the load histories of future aircraft are of prime interest, the aim of much of today's work is aimed at collecting data on the distrubance and operating conditions rather than collecting masses of load statistics on yesterday's designs.

The final principle guiding research in connection with repeated loads is to subdivide the work into studies of the distrubance, operating conditions, and airplane behavior. The study of airplane behavior seems to be the most formidable task of the three, since it involves extensive instrumentation and flight tests under controlled conditions. Other pertinent considerations are:

1. For gusts, the standard expression for the power spectrum, $\Phi(w)$ doesn't cover the larger wave lengths very well.

2. The selection of a single value of L assumes that the spectral shape for atmospheric turbulence is independent of source, an assumption requiring additional verification particularly for the turbulence of convective clouds.

3. Profiles for taxi loads in the form of power spectrumd indicate a rapid decrease in power as the wavelengths decrease. A major diffi-culty arises from the fact that shock struts appear to have a threshold below which they do not operate.

4. For commercial operations, the short-duration low-altitude flights characterizing feeder-line operations, gust loads consume about 80% of the airplane fatigue life. For high-altitude jet transports 60% of fatigue life can be ascribed to gusts. (Ref. 6)

VIII Some NASA Experiences in the Application of PSD to Maneuver and Taxi Loads

Application of PSD to maneuver loads obtained on jet fighters was investigated by Hamer.

It was determined that the maneuvering load-factor time histories appeared to be described by a truncated normal distribution. (Fig. 8)

The power spectral densities obtained were relatively level at frequencies below 0.03 cycle per second and varied inversely with approximately the cube of the frequency at the higher frequencies. In general, the frequency content was very low above 0.2 cycle per second. The load-factor peak distributions were estimated fairly well from the spectrum analysis. In addition, peak load data obtained during service operations of fighter-type airplanes with flight time totaling about 24,000 hours were examined and appeared to agree reasonably well with the type of equations obtained from peak-load distributions.

Also, the frequency content of some airplane response quantities obtained from a number of operational training flights of a fighter airplane was presented in another study by Hamer. Power spectral densities of normal and transverse load factor and pitching acceleration were shown for several types of missions normally performed by the aircraft. The frequency content, which is described by the spectrum, provides information which is useful in the design of recording and computing equipment for analyzing maneuver-load data.

When normalized by dividing the mean-square value, the results indicated that except for some differences at the higher frequencies due to the effect of rough air, the frequency content of each of the airplane response quantities was similar during the different types of mission investigated.

The normal-load-factor data for the different types of missions exhibited some of the characteristics of a Gaussian random process; therefore, power spectral methods were used in analyzing the maneuver-load factor data. Normal-load-factor peak distributions were estimated to a reasonable degree of accuracy from this spectrum analysis. Peak distributions for transverse load factor and pitching acceleration were not determined from the power spectrum because in most types of missions the quantities did not appear to have the characteristics of a Gaussian-random process.

Morris (Ref. 21) conducted a study of the response of a light aircraft to roughness of unpaved runways; and found that mean-wquare acceleration response increased approximately linearly up to about 25 or 30 mph., but increased more rapidly than the linear rate at higher speeds. Transfer functions computed from the air craft acceleration response to the runways were in relatively good agreement at 20 mph. As regards to the peak amplitude and frequency at which it occurred, apparent lack of agreement at 40 mph may have resulted from nonlinearities in the system and/or for small a statistical sample for determining a transfer function.

IX ASR/DS-11 Experiences in The Application of Powers Spectral Methods to Airplane Design under Project 1367 Structural Design Criteria

Study of World Wide Turbulence Data (Reference 22)

New York University is making an analysis of B-66B low level turbulence data to determine spectral variance and spectral slope behavior with respect to four physical parameter: Mean wind speed, atmospheric stability, terrain and height; and is re-examining the basic approach to the gust load design problem. Current data indicates that the normalized spectral shapes are sensibly independent of wind speed and stability. Additional analyses are necessary to confirm these spectral shape results.

Heating Effects on Gust Response (Reference 26)

The response of a high-speed flexible vehicle to atmospheric turbulence was investigated for flight at an altitude of 1000 ft. The Mach number range was from 0.7 to 6.0. The response of the structure at nominal room temperatures was compared with the response of the structure when aerodynamically heated to a uniform temperature. (Reference Figure 9) The response to discrete gusts was compared with the response to continuous random turbulence (computed by power spectral densities), and the effect of flexibility was investigated. Both methods used oscillatory pressure distributions. The results have not been tested and are applicable to similar configurations only. These results indicate that: (a) Rms acceleration and bending moment responses do not necessarily increase with speed continuously, (b) Maximum rms generally occur at between Mach 4 and Mach 5, except for fuselage bending moment which is slightly higher at Mach 6, (c) One degree of freedom underestimates the rms acceleration and bending moment responses with the exception of the fuselage bending moment in the Mach number range between Mach 3 and Mach 5, (d) The number of degrees of freedom have a considerable effect on the rms response. An extreme example is the rms nose acceleration at Mach 5. The five degrees of freedom rms response is approximately 330% higher than predicted by one degree of freedom.

Whereas, the discrete gust method has no means of predicting a change in the number of peaks exceeding the zero level due to heating, the continuous gust method does predict a change in the number of peaks exceeding the zero level through the change in the characteristic frequency. The rms response level of nose acceleration increases by approximately 62% due to heating. The ratio of the fuselage bending moment stress to the ultimate stress increases by approximately 75% due to heating. Over one-half of the latter increase is due to the reduction in ultimate stress produced by heating.

Optimum Fatigue Spectra (Reference 23)

Two types of power spectral density analyses of taxi loads were made. The first was an analog study in which the complete non-linear system of gear equations was considered. Runways were generated and the response of the system in the time domain to taxi over these runways was produced. Cumulative occurrences of incremental c.q. acceleration levels were then obtained by counting the number of crossings with positive slope of the

various levels. The second type of analysis comprised a digital study. It was the aim of the analysis to ascertain whether it was possible to establish the cumulative occurrences of incremental c.q. acceleration by conventional spectral analysis in the frequency domain with a suitable linearized representation on the main and nose gear forces. The airplane configuration used was the DC-7c at 143,000 lb gross weight. A single taxi velocity of 22 ft per second was considered throughout. The results of the digital analysis were found to be powerfully affected by the linearized representation of the oleo dampings. Linearization of the oleo damping requires the selection of an amplitude and frequency. In AGARD Report 119, it is suggested that the basis of selection should be a dominating strut amplitude and frequency. A new procedure was arrived as to obtain a more specific criteria for linearization of the oleo damping, having due regard to the stroke response at all frequencies. By way of comparison the non-linear results exhibit a definite bias toward the negative, or upward accelerations. For negative c.g. accelerations the agreement between linear and non-linear analysis was satisfactory. The most promising procedure seems to be to consider the non-linear gear with a stationary random input. The non-linear gear analyses also show that the resonances associated with the unsprung gear masses can have an appreciable effect on the cumulative occurrences of c.g. accelerations unless suitably filtered. These resonances ordinarily lie at frequencies beyond the range of definition of the runway roughness power spectra of NACA TN 4303. Oscillograph records obtained during airplane taxi conditions have shown high frequency content, indicating excitation of mode lying outside the range of definition. It would therefore seem desirable to extend the bandwidth of the spectra of NACA TN 4303 to higher frequencies. It should be re-emphasized that the above results were based on one taxi velocity. There is a lack of experimental data on the distribution of taxi velocity. Moreover, the direct application of runway data in spectral form to fatigue studies is complicated by the non-linear transmission characteristics of landing gears in current usage. Other phases of taxi loads such as turning, pivoting, braking and towing should be considered in fatigue analysis. Unfortunately, the only data available on these are from specific flight test programs conducted to verify design criteria, in the traditional sense; not fatigue criteria. Much analytical insight has been gained by investigations to date, however.

B-66 Low-Level Gust Study (Reference 31)

Statistical design, data gathering, instrumentation, data reduction, computation, and presentation were accomplished. Technical analysis of the data was performed, i.e., determination of the scale of turbulence, numerical filtering, integration, resolution and spectral estimation. Comparison of current Mil Specs on low-altitude gusts and response equations with sampled data results, recommendations for future analysis and flight programs. Comparison with Mil Spec 8866 and NACA TN 4332 showed that sample distributions of standard deviation of low-level gusts differ significantly, equations in specifications overestimate the number of most frequently occurring gust peaks, which are of interest in repeated loads, and underestimate the number of large gust peaks. The technical analysis further showed that: Extermal procedures by Gumbel are very

useful in relating expected gust peaks to miles of flight. Isotropic
turbulence approximations are excellent fits to auto-correlation and
power spectrum of low-level gusts. The Chi-square distribution is a
significant fit to sample distributions. Sample derived equivalent gust
velocities are good (not tested) estimates of observed gust velocities.
Cross-spectral power between sample gust velocity components in negligible;
phase is uncorrelated; amplitude is uncorrelated. Transfer functions by
cross spectral procedures are good (not tested) estimates of analytical
transfer functions. Statistical techniques such as, cross correlation,
sampling, regression, product moment correlation coefficients, etc.;
are useful tools in separating gust and maneuver responses; also the
distribution of gust lengths is useful in separating gust and maneuver
responses. Plans are currently being made to gather similar data for
the summer months to arrive at a more comprehensive gust model.

X Relating Peak Count Data to RMS Data

In regards to techniques for relating peak count data to rms data
for low-level flight, Notess (Reference 24) concludes that these two
types of data (peak count and rms) can be related in a way which
enables a simple application to the specific problem at hand. Data
from "Notess, Analyses of Turbulence Data Measured in Flight at
Altitudes up to 1600 ft above Three Different Types of Terrain,"
and CAL Report Nr TE-1215-F-1, Feburary 1959 and WADD TR 60-305,
"B-66B Low Level Gust Study," was analyzed and indicates that on
the average a gust of extreme magnitude is usually associated with
a nearly homogeneous patch of turbulence having an rms of one-fourth
the extreme value in an extent of ten miles. Thus, it is reasonable
to assume that in every flight there will be at least one patch of
turbulence (10 miles) which is associated with the peak gust encountered
and will have an rms gust velocity equal to one-fourth of this peak
value. One peak exceeds four σ every ten miles. This is based on 40
data runs (from B-66 data) wherein the peak variation for 35 of the
runs was between 3.4 σ to 5.0 σ .

Considerable work remains to be done so that a workable model can
be obtained which will enable one to take into account the relationship
of the intensity levels among adjacent turbulence patches.

Existing turbulence models such as NACA TR 1272 do not provide data
on these interrelationships since peak count data for many flights is
lumped together into one probability distribution. (Figure 9)

This can be done for design purposes to some extent if one applies
the probabilities to missions rather than to individual ten mile patches.
In other words, one should say one out of seven missions will be performed
in turbulence of five σ rather than one out of seven patches in a given
mission.

XI Applied Research Objective - 35A55

Recognizing that the reluctance to use power spectral methods is mostly due to the complexity of the analytical techniques, and the fact that the techniques of statistical analysis are often unfamiliar to designers, it is incumbent upon the Air Force to pursue investigation aimed toward simplifying and applying power spectral techniques. Toward this end, Applied Research Objective Nr 35A55 was established.

Input and Response Power Spectrum of Random Load Inputs to Advanced Vehicles

OBJECTIVE: To determine the spectral shape and intensity of the input and response power spectrum of advanced vehicle structures resulting from gusts, ground winds, wind shears, pilot inputs, automatic guidance, and maneuvers, in order to apply these spectra and root-mean square distributions in structural load calculations in which the effect of vehicle motions and flexibility are included in the determination of vehicle transfer functions which can be converted to real time differential equations solvable on high-speed computers.

PRESENT STATUS: The spectral shape of gusts has been adequately determined. Work is continuing on the variation of intensity with geographic and meteorological factors which influence gusts. Input power spectrum and response power spectrum are being measured for ground winds, wind shears, pilot inputs, and maneuvers. No methods are currently available for converting available data from its present form into a form appropriate for spectral-type calculations.

TECHNICAL POSSIBILITIES: Further analytical investigations to determine methods for re-evaluating currently available data on winds, wind shears, pilot inputs, automatic guidance systems, and maneuvers from their present form into a form appropriate for spectral calculations as has been done for gusts in NACA TR 1272. Further analytical investigations to develop a mathematical model representative of the spectral shape of wind shear, pilot, maneuver, automatic guidance system input. Further analytical investigations to determine goodness of fit criterion for sample distributions of this data. Further analytical investigations to determine extremal procedures in relating expected peaks to miles of flight for inputs other than gusts. Further investigation to determine the significance of cross-spectral power between sample wind and wind shear of cross-spectral power between sample wind and wind shear components. Further investigation to determine how well transfer functions. Further investigation to determine how useful statistical technique such as cross-correlation, sampling regression, product moment correlation, coefficients, etc., are in separating responses from the aforementioned inputs.

<u>GOVERNMENT AGENCIES DOING WORK IN THE AREA</u>: The Navy is directing work in the area of determining the spectral shape and intensity of the input and response power spectrum of flight vehicles due to maneuvers and pilot inputs.

XII Concluding Remarks

Neither this report nor the applied research objective presumes to encompass all new concepts which if successfully pursued, would enhance our knowledge and advance the-state-of-the-art.

In fact, it is hoped that this report serves to discover and stimulate investigation and presents some methods of demonstration which tend to lead the reader to investigate further.

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APPENDIX I   FIGURES

Figure I
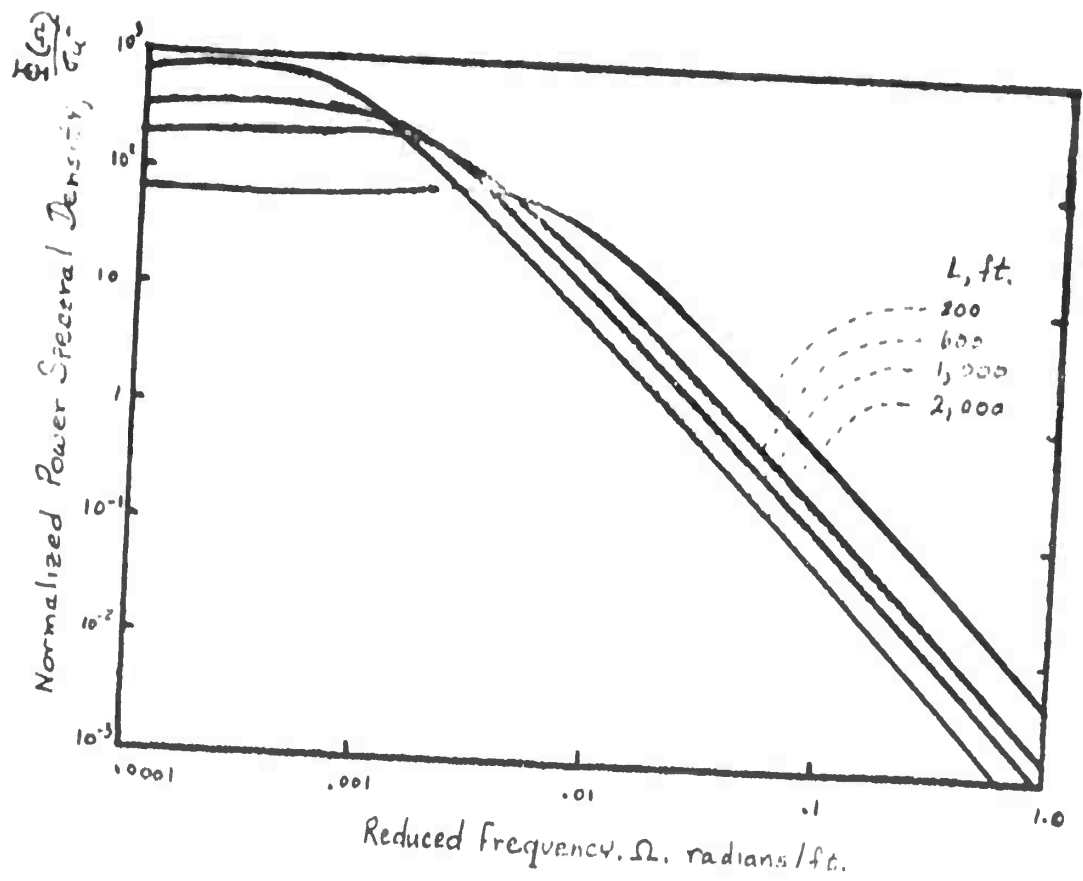
Analytic representation of the spectrum of atmospheric turbulence[*]:

$$\Phi(\Omega) = \sigma_u^2 \frac{L}{\pi} \frac{1 + 3\Omega^2 L^2}{(1 + \Omega^2 L^2)^2}$$

Ref. — NACA TR 1272

Figure 2a

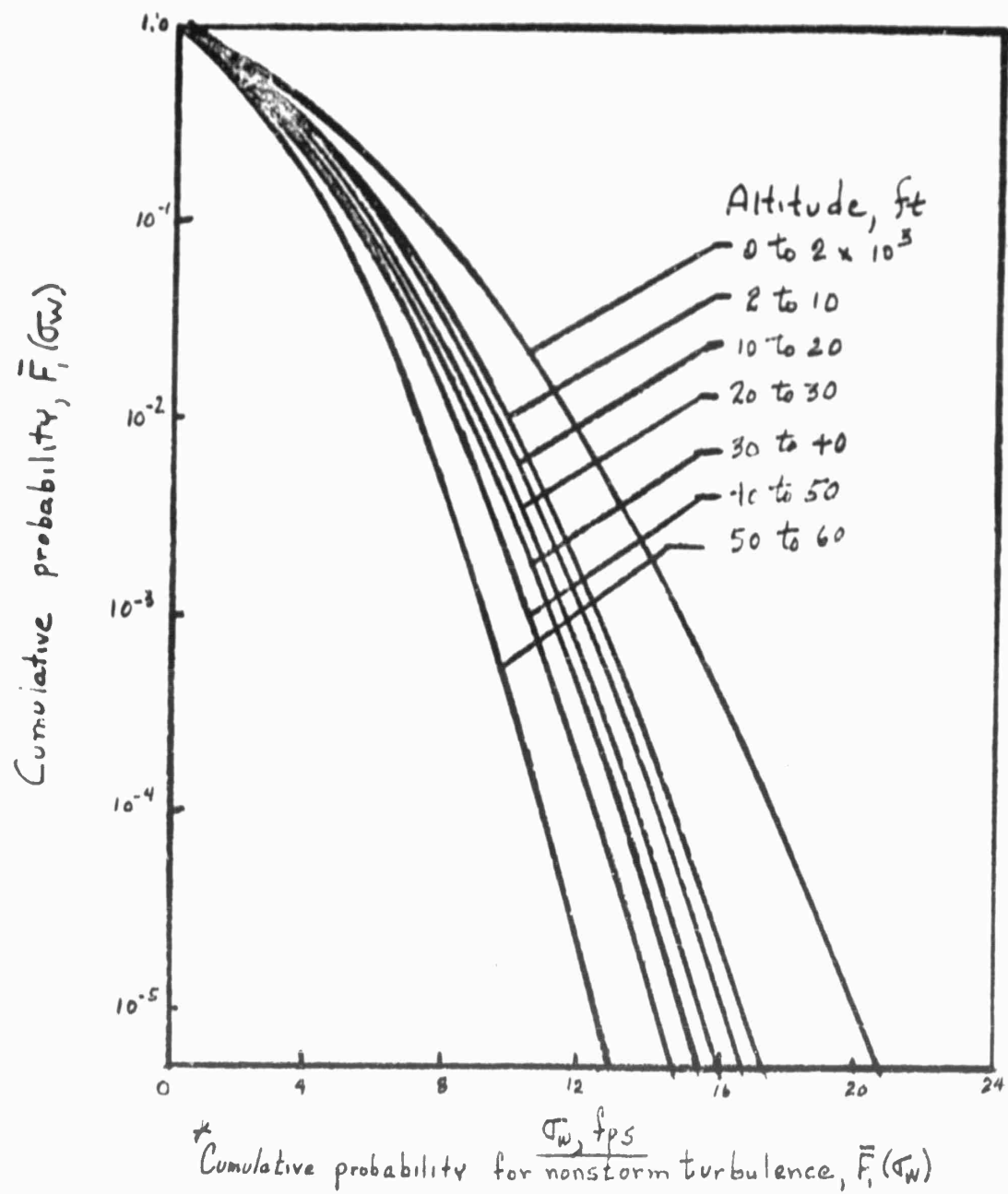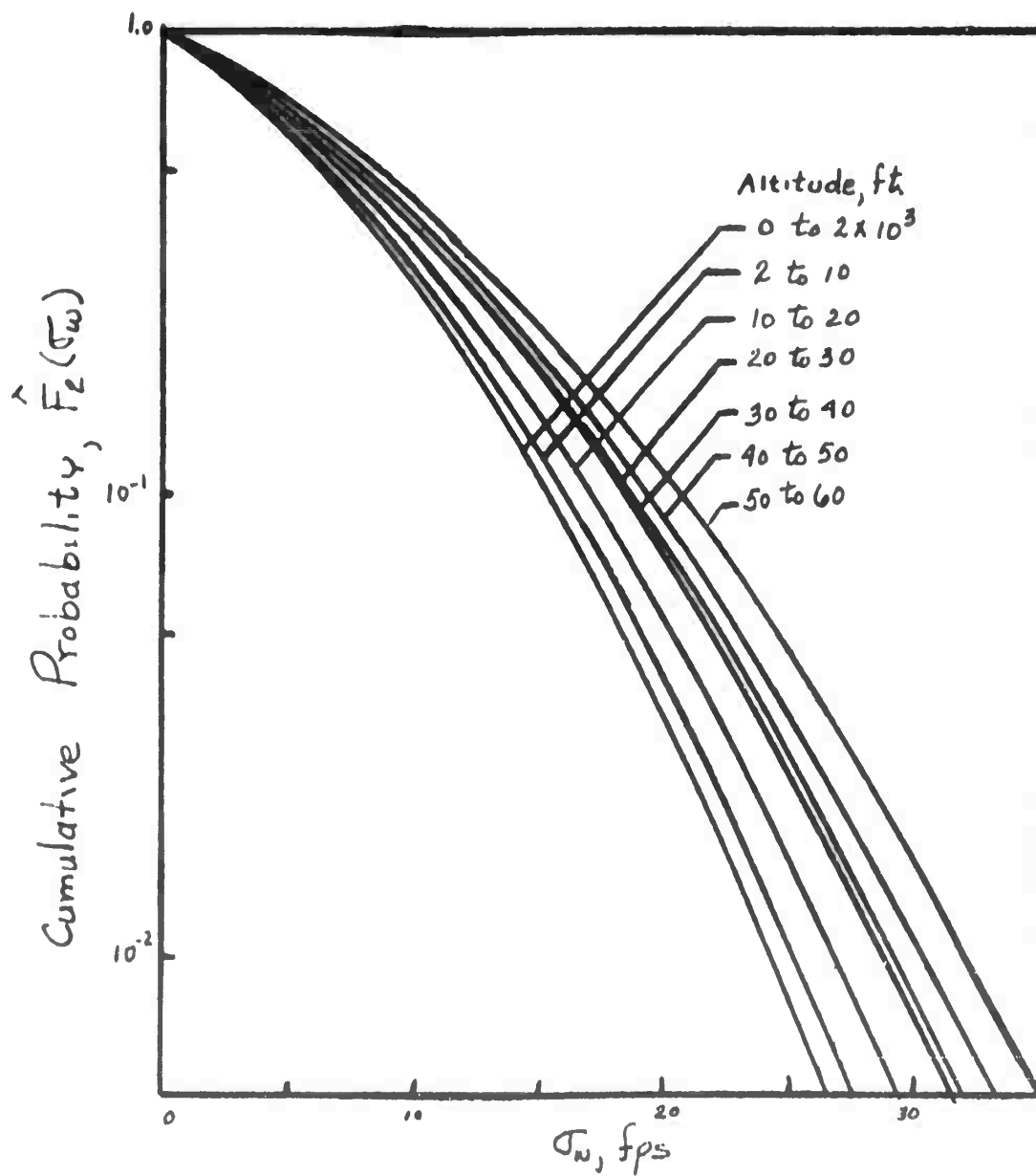Cumulative probability for nonstorm turbulence, $\bar{F}_1(\sigma_w)$

Figure 2b

Cumulative probability for storm turbulence

* Ref. NACA TN 4332.
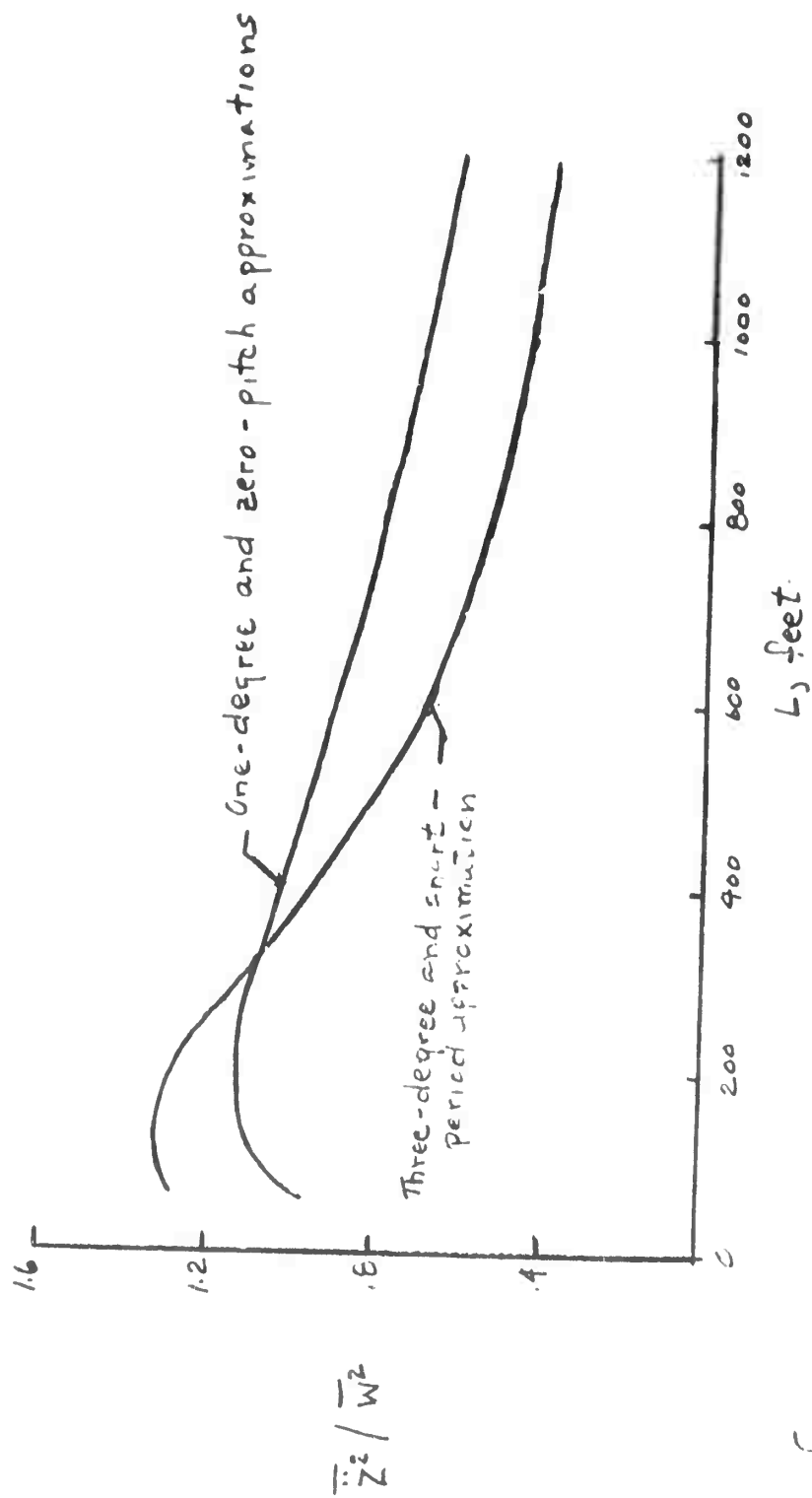
Figure 3

Figure 3. Effect of scale of turbulence on the mean-square acceleration of an example airplane. (Ref. No. 7)

Figure 4   Appendix I
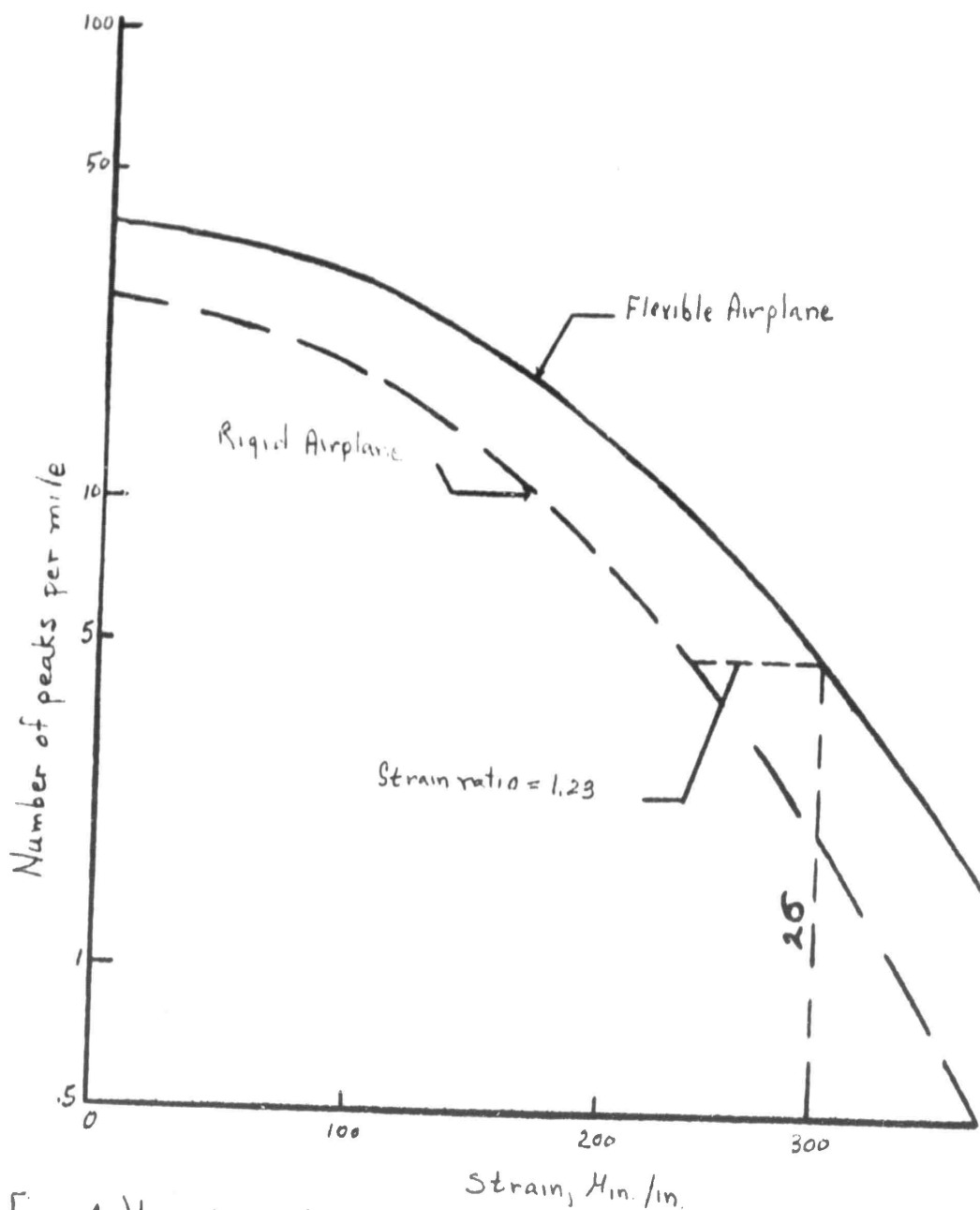

Figure 4. Variation of the number of strain peaks with strain Level for the rigid and flexible conditions at one station. (Ref. No. 32)

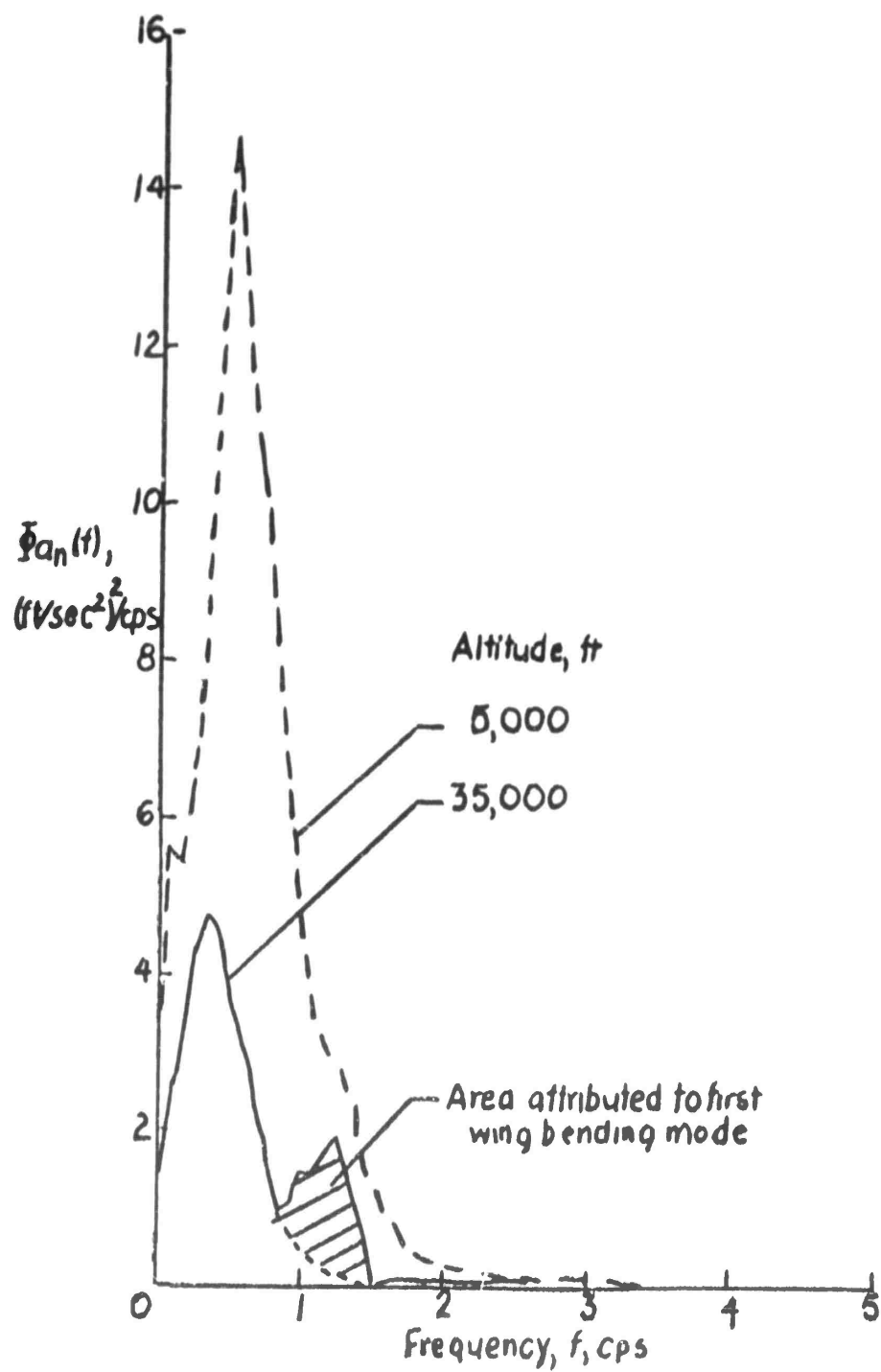Figure 5   Appendix I

Figure 5, Comparison of power spectrum of faired center-of-gravity acceleration for 5,000 feet and 35,000 altitude. (Ref. No. 29)
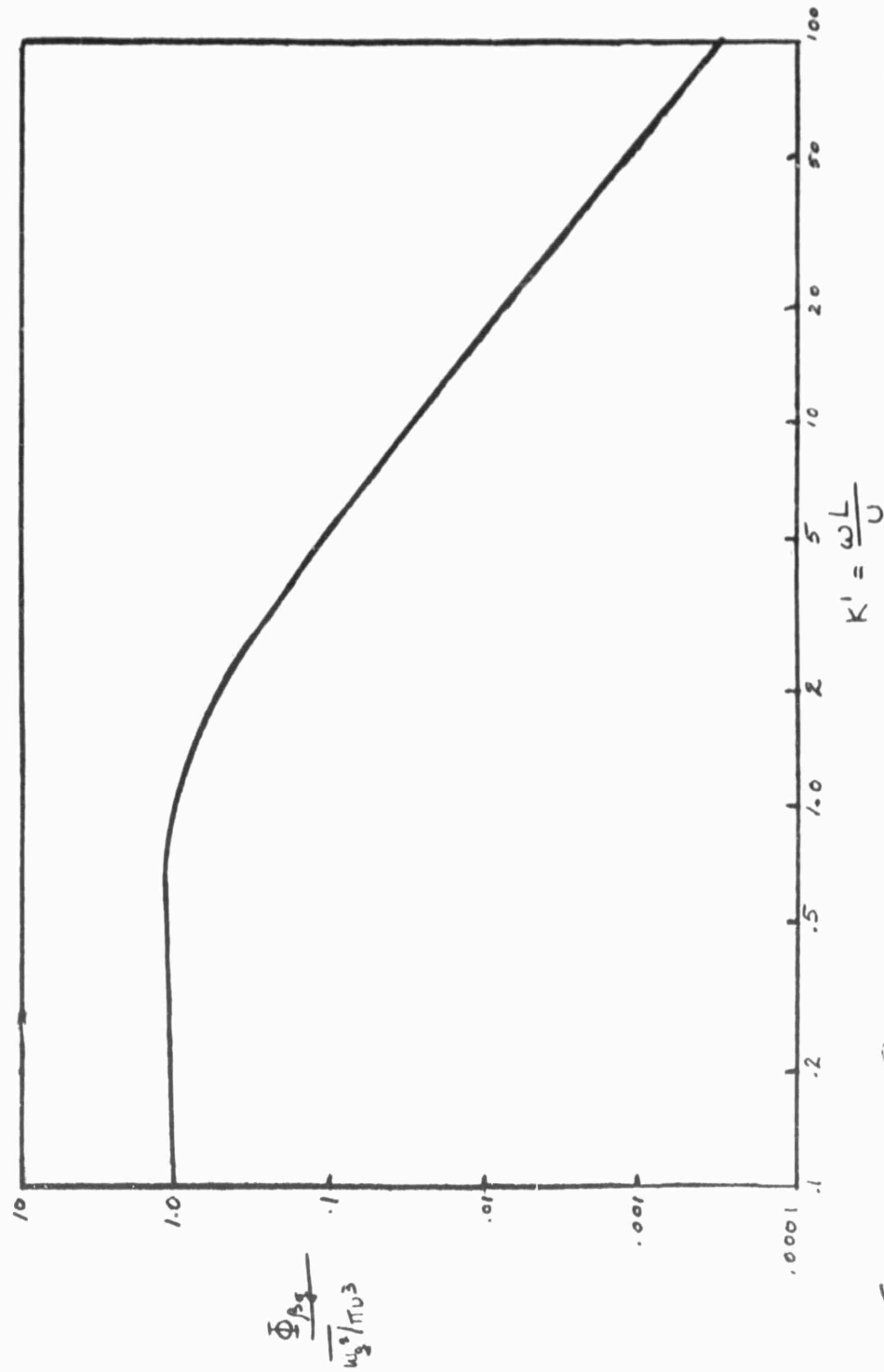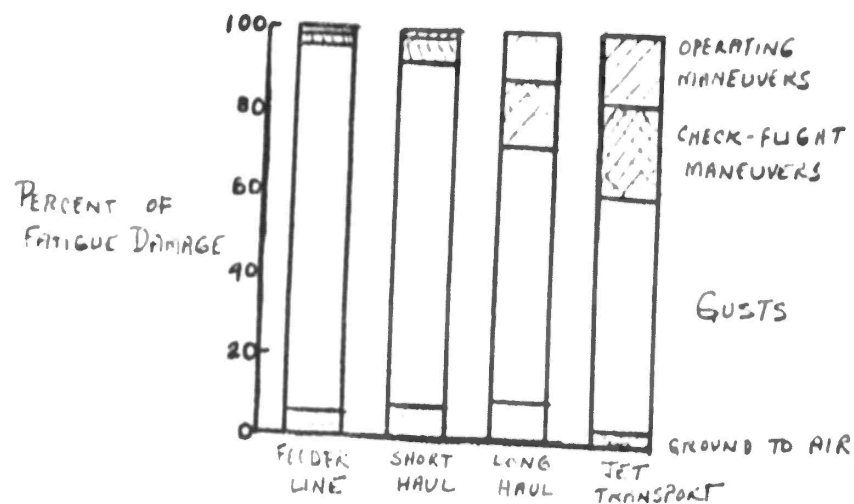
Figure 6

Figure 6. Power spectral density of the side-gust component of isotropic turbulence (Ref. No. 8)
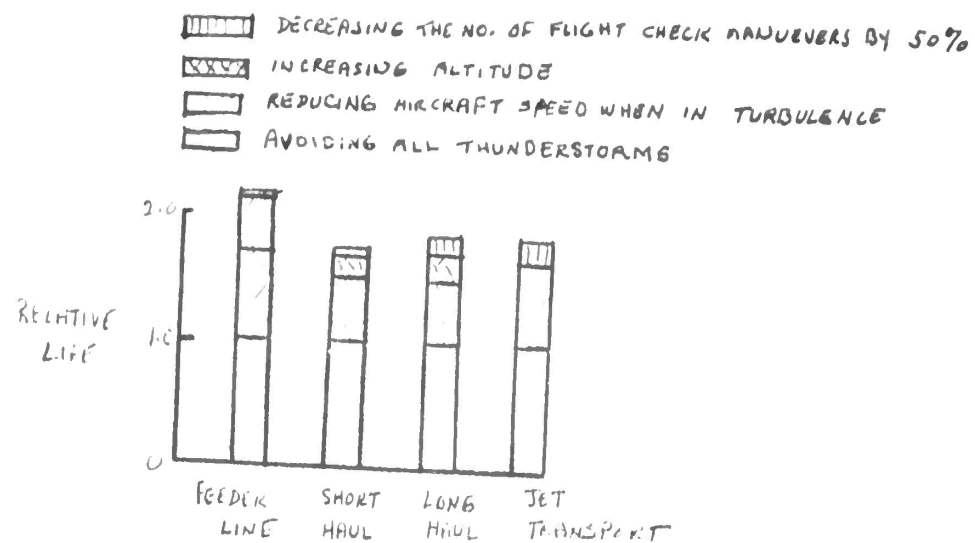
Figure 7   Appendix I

A. FATIGUE DAMAGE ATTRIBUTED TO VARIOUS LOAD SOURCES

B. INCREASE IN FATIGUE LIFE DUE TO SOME CHANGES IN
         OPERATING PRACTICES   (Res. No. 6)

Figure 7

Figure 10

LOW ALTITUDE GUST PROBABILITIES

- - - NACA TN 4332 0-2000 FT.
——— NACA TR 1272 0-10,000 FT.
●●● MIL A 8866 0-1000 FT.
✶✶ ESTIMATED $\sigma_{w_g} \approx U_{de}/4$
▲▲ FAIRED AVERAGE

CUMULATIVE PRO. BILITY OF EXCEEDING σ_{w_g}
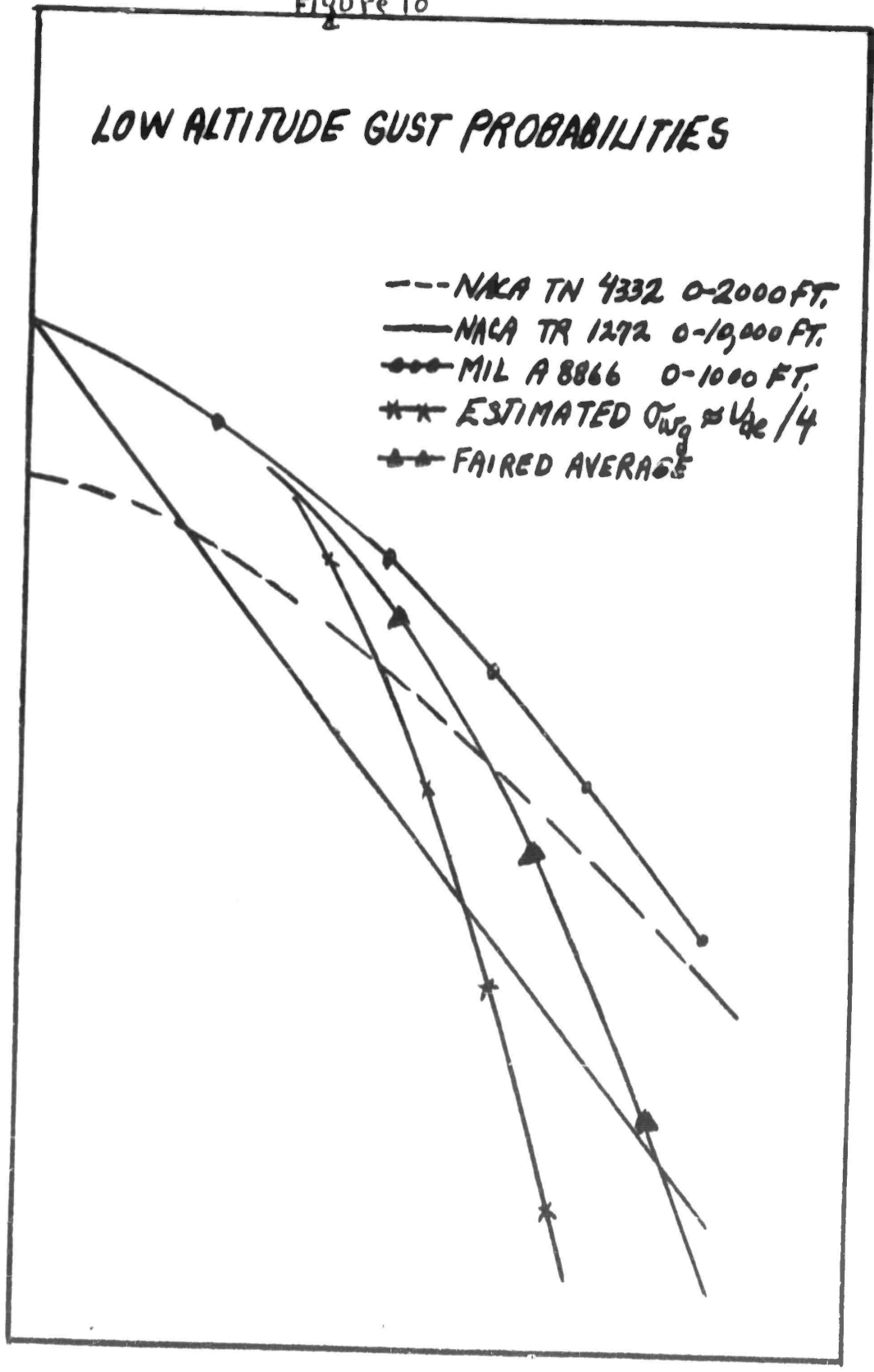
RMS GUST VELOCITY σ_{w_g} FPS

APPENDIX II

INTRODUCTION TO SPECTRAL ANALYSIS

In a very real sense Power Spectral Density is nothing new. Since we are again confronted with the usual engineering problem of trying to discover a pattern which coupled with ingenuity, will enable us to decide what will happen in time (or distance) from what has happened.

Since the most obvious form of repetition is running around a circle, we shall express all predictions in terms of the framework of periodicity or frequency and amplitude,

$$y = \alpha \sin wt$$

In other words, we have decided to use the principles of harmonic analysis.

Moreover, in observing actual records of a continuous disturbance such as gusts, we soon recognize that we are dealing with the problem of performing an amplitude-frequency analysis of a random time history which is neither periodic (repetitive) nor decaying with time.

Thus like it or not, there is no choice but to overcome the natural desire for "something nailed down," and to speak the language of probability instead of the language of certainty.

For if a time history is random, it must have elements of uncertainty and all one can hope to determine is the degree of uncertainty.

In view of the above, some basic statistical terms will now be presented.

STATISTICAL TERMINOLOGY

MEAN OR AVERAGE:

Let t be a real variable, and y = f(T) be any function given on an interval (a,b). Then $\bar{y}$, the average of y with respect to t for the interval a,b is defined by
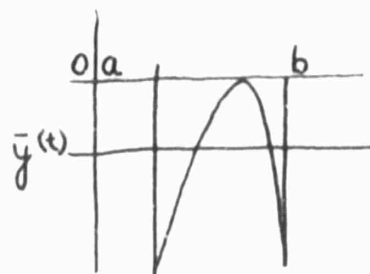
$$\bar{y} = \frac{1}{b-a} \int_a^b y \, dt = \frac{1}{b-a} \int_a^b f(t) \, dt$$

or

$$(b-a) \, \bar{y} = \int_a^b y \, dt$$

The geometric interpretation makes it easy to remember the definition:
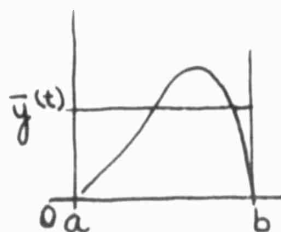
for positive $\bar{y}(t)$

for negative $\bar{y}(t)$

ROOT-MEAN-SQUARE   (RMS)

We define $\bar{\bar{y}}$, the root mean square (rms) value of y with respect to t for the interval a,b by the equation:

$$\bar{\bar{y}} = rms\, y = \sqrt{\frac{\int_a^b y^2 dt}{b-a}}$$

Thus the rms $\bar{\bar{y}}$ is the square root of the average of the square of y, and is so defined that

$$(b-a)(\bar{\bar{y}})^2 = \int_a^b y^2\, dt$$

This definition, also becomes clearer with an example:

Consider $y = 10t^m, m > 0$, on the interval 0,1. Then

$$\int_0^1 (10t^m)^2 dt = \frac{100}{2m+1} t^{2m+1} \Big]_0^1 = \frac{100}{2m+1}$$

so that in this case

$$\bar{\bar{y}} = \sqrt{\frac{1}{1-0}\frac{100}{2m+1}} = \frac{10}{\sqrt{2m+1}}$$

And if we take special values of m,

FOR m	999	4	1	0.25	$\frac{1}{999}$
$\bar{\bar{y}}$	0.224	3.333	5.773	8.166	9.99001

The extreme values of $y(t) = 10t^m$ on 0,1 are 0 and 10. This illustrates that the rms value of y always lies between the extreme values of $y(t)$ but may lie quite close to either.

AUTOCORRELATION FUNCTIONS

The easiest quantitative way to judge the correlation or fit between two things is by their product. In general, large products mean good correlation and small products mean poor correlation. When the "things" are dependent on time, say $f_1(t)$ and $f_2(t)$. [$f_2(t)$ could be equal to $f(t_1 + \tau)$]) one takes the average product over all the time involved. Thus the autocorrelation of a time history is

$$A(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t)\, f(t + \tau)\, dt$$

ELEMENTS OF TRANSFORMATION THEORY

Returning to the problem of the amplitude-frequency analysis of a time-history; it may be either periodic or not. If strictly periodic it can usually be expressed as a Fourier Series.

$$f(t) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(\omega_n) e^{i\omega_n t} dt$$

where
$$F(\omega_n) = \int_{T/2}^{T} f(t) e^{i\omega_n t} dt$$

(Notice how we now have related time, t, to frequency, ω, and vice versa.)

If f(t) is not strictly periodic, it may either settle to some constant value as time goes on or it may not.

If f(t) is non-periodic but decays to zero as time increases it can (usually) be expressed by a slight generalization of Fourier Series using continuous rather than discrete frequencies.

ELEMENTS OF TRANSFORMATION THEORY

This transformation from a time function to a frequency function is expressed by the Fourier integral

$$G(j\omega) = \int_{-\infty}^{\infty} F(t) e^{j\omega t} dt \qquad (1)$$

For any function $F(t) = 0, t > 0,$ the limits of integration can be changed and the transform restated as

$$G(j\omega) = \int_{0}^{\infty} F(t) e^{-j\omega t} dt \qquad (2)$$

making use of the Euler relationship:

$$e^{-j\omega t} = \cos \omega t - j \sin \omega t \qquad (3)$$

$G(j\omega)$ can be separated into components

$$A_\omega \cong Q[G(j\omega)] = \int_{0}^{\infty} F(t) \cos \omega t \, dt \qquad (4)$$

$$B_\omega \cong I[G(j\omega)] = \int_{0}^{\infty} F(t) \sin \omega t \, dt \qquad (5)$$

In polar coordinates $G(j\omega) = r e^{j\phi}$ $\qquad (6)$

where $\qquad (7)$

$$r = \sqrt{A_\omega^2 + B_\omega^2}$$

$$\phi = \arctan \frac{-B_\omega}{A_\omega} \qquad (8)$$

The practical problem is to determine a mathematical representation for $G(j\omega)$ such that the integrals (4) and (5) can be evaluated. Again, the question as to whether $\int_{0}^{\infty} F(t) e^{j\omega t}$ exists can be answered affirmatively for any $F(t)$ describing a physically realizable motion.

More pertinent is the question: Can the function be approximated by some $F_1(t)$ such that

$$\int_{0}^{\infty} F(t) e^{-j\omega t} dt \cong F_I(t) e^{-j\omega t} dt \qquad (9)$$

Equation (9) requires that $F(t)$ becomes zero, or nearly so, after some finite time T. If this condition cannot be met it is still possible to approximate (2) be breaking up $F(t)$ into parts

$$\int_0^\infty F(t)e^{-j\omega t}\,dt \approx \int_0^T F_I(t)e^{-j\omega t} + \int_T^\infty F_R(t)e^{-j\omega t}\,dt$$

where $F_R(t)$ is such that the second integral on the right can be formally evaluated.

The next logical question is: "If these transforms are so easy to evaluate, why did we skip over that part?"

So to that end, an intuitive description which is consistent with the level of this report is shown below. This approach was selected to avoid the "forest-for-the-trees" enigma.

COMPUTATIONAL TECHNIQUES FOR FOURIER TRANSFORMATION

Given a function (of time), f(t), its Fourier Transform $F(\omega)$ (a function of frequency) is defined

$$F(\omega) \int_{-\infty}^{\infty} f(t) e^{-j\omega t} \, dt$$

where $j^2 = -1$ and e is the exponential function. We shall use the identity

$$e^{-j\omega t} = \cos \omega t - j \sin \omega t$$

and shall consider sine and Cosine Fourier Transforms $\int_{-\infty}^{\infty} f(t) \sin \omega t \, dt$

and $\int_{-\infty}^{\infty} f(t) \cos \omega t \, dt$

For much of our discussion, we consider only the Cosine Transforms since a parallel discussion would apply to the Sine Transform. Furthermore, since mathematically the time and frequency domains are interchangeable, we shall sometimes refer to $F(\omega)$ as the Spectrum in order to emphasize that it is a function of frequency. This usage arose because the motivation for this study came from the problem of obtaining PSD from Autocorrelation Functions.

Let us assume that f(t) is an even function; i.e., f(t) = f(-t). We consider each frequency, and ask how much relative contribution a cosine of that frequency makes. We consider f(t) to be made up of a number of component frequencies (actually a continum of frequencies), and we wish to determine the relative amptitudes associated with the frequencies.

The way in which the cosine transform answers this question may be described as shown in Figure 1. Given f(t), we choose some frequency, ω, and compare $\cos \omega t$ with f(t). In order to make the comparison, we form the product f(t) $\cos \omega t$. This product will be positive when f(t) and $\cos \omega t$ are both positive or both negative; and its values, positive or negative, give an instantaneous picture of whether f(t) and $\cos \omega t$ agree. But we want to compare f(t) and $\cos \omega t$ throughout their whole range, so we integrate the product f(t)$\cos \omega t$. As ω is varied, this process, which gives a step-by-step interpretation of the definition of the Cosine Transform, will give a relative measure of how much contribution each frequency makes to f(t). When we consider the process step-by-step in this fashion, we shall refer to ω as the scanning frequency.

Given an even function f(t)-
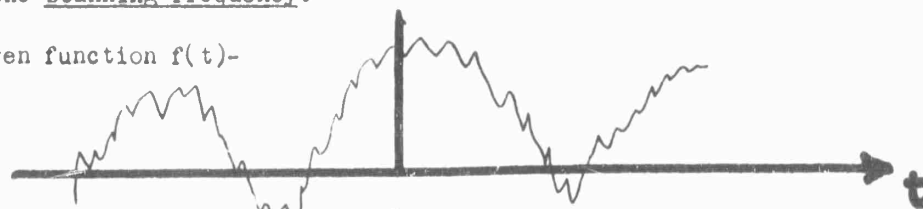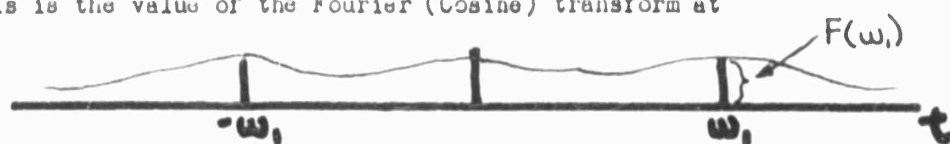

Pick a scanning frequency ω_{rr}
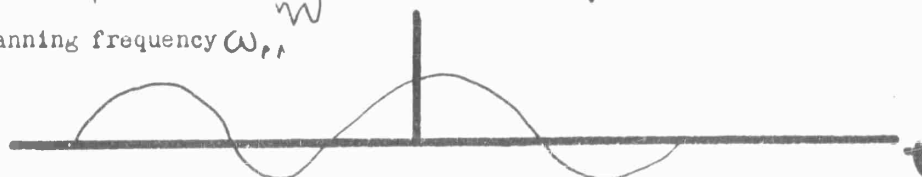
Form the instantaneous measure of agreement-$f(t) \cos \omega_1 t$

Integrate to obtain the total measure of agreement- **AREA = A**

$$A = \int_{-\infty}^{\infty} f(t) \cos \omega_1 t \, dt = F(\omega_1)$$

This is the value of the Fourier (Cosine) transform at

Since $f(t)$ and $\cos \omega t$ are even, $F(-\omega_1) = F(\omega_1)$.

Now to tie together some of the topics just discussed.

Thus far, in our effort to develop an amplitude-frequency analysis for a random time history, we have:

1) Discussed the autocorrelation function which describes the average value of the product between values of the record a specified time interval apart as a function of this time interval itself; and therefore gives the desired amplitude characterization of the random-time history, $f(t)$.

2) We have discussed the Fourier Integral Transformation which provides a frequency characterization of the random-time history.

We now should be able to analyse input-transfer-response for random functions by expressing input and response as Fourier Integrals of the autocorrelations of their respective time histories. In any case, we can implement this "tongue-twister" with the aid of the following outline of the actual mechanics normally involved:

THE MECHANICS OF COMPUTING THE POWER SPECTRA

1. Recording time sampled input data on magnetic tape.

2. Feeding this data into a large high-speed computer.

3. Computing the input spectra from this data.

4. Computing the transfer function for the system.

    (If the input and output spectra are known the transfer function is

$$\sqrt{\frac{\text{output spectra}}{\text{input spectra}}}$$

5. Recording the output.

6. Computing the response spectra.

<u>THE TUKEY PROCEDURE FOR COMPUTING POWER SPECTRA, $\Phi(\omega)$</u>
<u>ON A HIGH-SPEED COMPUTER</u>

For a finite record length sampled at t, the following steps may be used in computing gust spectra: (although other methods are available)

1. Calculate the mean, $\bar{y}(t)$,

$$\bar{y}(t) = \frac{1}{N} \sum_{i=1}^{N} y_i(t)$$

where N = # of values of the random variable taken at equal units of time, Δt, apart.

2. Calculate the autocorrelation coefficients,

$$R_\lambda = \frac{1}{n-\lambda} \sum_{q=1}^{n-\lambda} y_{qi} \, y_{qi+\lambda} \qquad \left(\begin{array}{l} \lambda = 0,1,2...,m \\ m+1 \ \text{VALUES} \end{array} \right)$$

where M = # of lags or uniformly spaced points over the frequency range at which estimates are desired
and

$$y_q(t) = y_i(t) - \bar{y}(t)$$

3. Calculate the initial estimates of power spectra

$$L_\lambda = \frac{\Delta t}{\pi} \left(R_0 + 2 \sum_{q=1}^{m-1} R \cos \frac{q \lambda \pi}{m} + R_m \cos \lambda \pi \right)$$

Calculate $m+1$ values of L_λ' $(R_0 \ \text{To} \ R_m)$

4. Calculate final estimates of power spectra

$$\Phi_0 = 0.54 \, L_0 + 0.46 \, L_1$$

$$\Phi_\lambda = 0.23 \, L_{\lambda-1} + 0.54 \, L_\lambda + 0.23 \, L_{\lambda+1}$$

$$\Phi_m = 0.46 \, L_{m-1} + 0.54 \, L_m$$

Calculate $m+1$ values of Φ_λ

These power spectral calculations may be quickly obtained by a high-speed digital computer.

The purpose of this section is to illustrate the practical aspects of the aforementioned topics.

If a random time history is considered as being analogous to current flowing through a unit resistance, an important characteristic is the power or energy contained in $f(t)$. This power may be conceived as composed of contributions from various frequencies. One speaks of the density of power or energy as the limit of the power in a frequency band as the bandwidth approaches zero. This plot of power against frequency is appropriately called the PSD curve or power spectrum. By knowledge of the transfer system, the input power spectrum can usually be converted to the output or response power spectrum, as shown above.

In order to develop a frequency representation which would be applicable to continuing disturbances, the theory of random processes makes use of the concept of a stationary random process. The basic assumption characterizing a stationary random process is that the underlying mechanism which gives rise to the disturbance are invariant with time and statistical prediction becomes possible.

For the case of a stationary random function of time $y(t)$, the mean square, $y^2(t)$ is defined by the following equation:

$$\overline{y^2(t)} = \lim_{K \to \infty} \int_0^T [y(t)]^2 \, dt \tag{1}$$

The mean square will usually exist and represent a measure of disturbance intensity. Since $y^2(t)$ is a quadratic function of $y(t)$, it has been termed the "average power" of $y^2(t)$ in analogy to electrical power which is proportional to the square of the current. The function $y(t)$ is considered to be composed of a infinite number of sinusoidal components with circular frequencies, ω, between 0 and infinity. The portion of $y^2(t)$ arising from components having frequencies between ω and $\omega + d\omega$ is denoted by $\Phi(\omega) d\omega$. The function $\Phi(\omega)$ is called the PSD function, and has the property that

$$\overline{y^2(t)} = \int_0^\infty \Phi(\omega) \, d\omega \tag{2}$$

The PSD function is therefore:

$$\Phi(\omega) = \lim_{K \to \infty} \frac{1}{\pi \tau} \left| \int_0^T y(t) e^{-i\omega t} \, dt \right|^2 \tag{3}$$

Equation (3) may be used to evaluate the PSD function from observed data. However, in practice, the PSD function may be more conveniently determined using a related function, the Autocorrelation function, $R(\tau)$, defined by

$$R(\tau) = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\infty y(t) y(t+\tau) \, dt$$

The autocorrelation function has the symmetrical property $R(\tau)=R(-\tau)$ and is reciprocally related to the PSD function by the Fourier Cosine Transformation in the following manner:

$$\begin{cases} R(\tau) = \int_0^\infty \Phi(\omega) \cos \omega\tau \, d\omega \\ \Phi(\omega) = 2/\pi \int_0^\infty R(\tau) \cos \omega\tau \, d\tau \end{cases}$$

This definition of the PSD function is consistent with the preceding definition of equation (3). See Appendix - Computational Techniques for Fourier Transformation.

When a linear system is exposed to an input varying in a random manner with time, the probability distribution of the system output y can frequently be represented by a normal probability density distribution defined by the relation (5) where y and are the mean and standard deviation and are defined by (6):

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(y-\bar{y}/\sigma)^2} \quad (5)$$

$$\left. \begin{aligned} \bar{y} &= \lim_{T \to \infty} \frac{1}{T} \int_0^T y \, dt \\ \sigma &= \left[\lim_{T \to \infty} \int_0^T (y-\bar{y})^2 \, dt\right]^{1/2} \end{aligned} \right\} \quad (6)$$

Investigations of communication problems associated with random noise, which has many obvious similarities to turbulence, have shown that normal distributions are frequently encountered. Rice, in his "Math. Anal. of Random Noise" has shown that for a linear system the shot effect in a vacuum tube gives rise to a noise current which has a normal distribution of current intensity. Investigation of fluid turbulence frequently yields normal distributions of velocity fluctuations.

Statistically, whenever certain conditions are met, the distribution in question can be considered as eighther a normal distribution, of some skew distribution which can be transformed into a function of a normal distribution. When these conditions are met standard methods are available for testing the validity of all statistical hypotheses and assumptions.

Measurements indicate that over the frequency range of interest the power spectrum of atmospheric turbulence may be well approximated by simple analytic expressions such as have been used in wind tunnel studies of isotropic turbulence. Also, in the theory of random processes, relations have been derived between peak counts and the associated power spectra. These relations apply to the case of a stationary Gaussian or Normal random process, where the term Gaussian designates a process characterized by a Gaussian distribution for the amplitude of the disturbances as well as for its time derivatives. Actual results support such as assumption.

Since the intensity of turbulence varies with weather conditions, the operational gust history is considered to be a nonstationary Gaussian process varying only in intensity or rms gust velocity; and the problem of specifying the gust history is reduced to that of specifying the probability distribution of the rms gust velocity.

CUMULATIVE PROBABILITY DISTRIBUTION OF rms GUST VELOCITY
NASA VS. USAF DATA

On the basis of these considerations, techniques have been developed for the estimation of the probability distributions of the rms acceleration and the rms gust velocity from data on peak accelerations. These basic techniques developed by Dr. Harry Press, and associates at NASA were applied to available peak count statistics from B-47, B-52, and KC-135 aircraft and the associated probability distributions of root mean square gust velocity are derived.

Limitations and application of these results to calculation of gust-load and other airplane response histories: (See Figure 1)

1. This graph represents 764.2 hours of B-47, B-52, and KC-135 flight time above 10,000 feet. In determining the transfer functions, average values of airplane and operational characteristics were used. The broken curve is superimposed as a basis of comparison. It represents NACA's results in transport-type operations from 30,000 - 50,000 feet.

2. The cumulative probability distributions, shown were obtained by integrating the probability distributions, and define the proportion of total flight time spent in turbulence exceeding given values of σ_u.

3. The description of the gust experience in this form is directly applicable to load calculations for other airplanes in similar operations by reversing the procedures used in obtaining these results. However, direct application of these results would apply only to similar operations. In order to obtain results that are more flexible and applicable to arbitrary flight plans, it would be desirable to determine the variations in these distributions with altitude, weather condition, and perhaps geography.
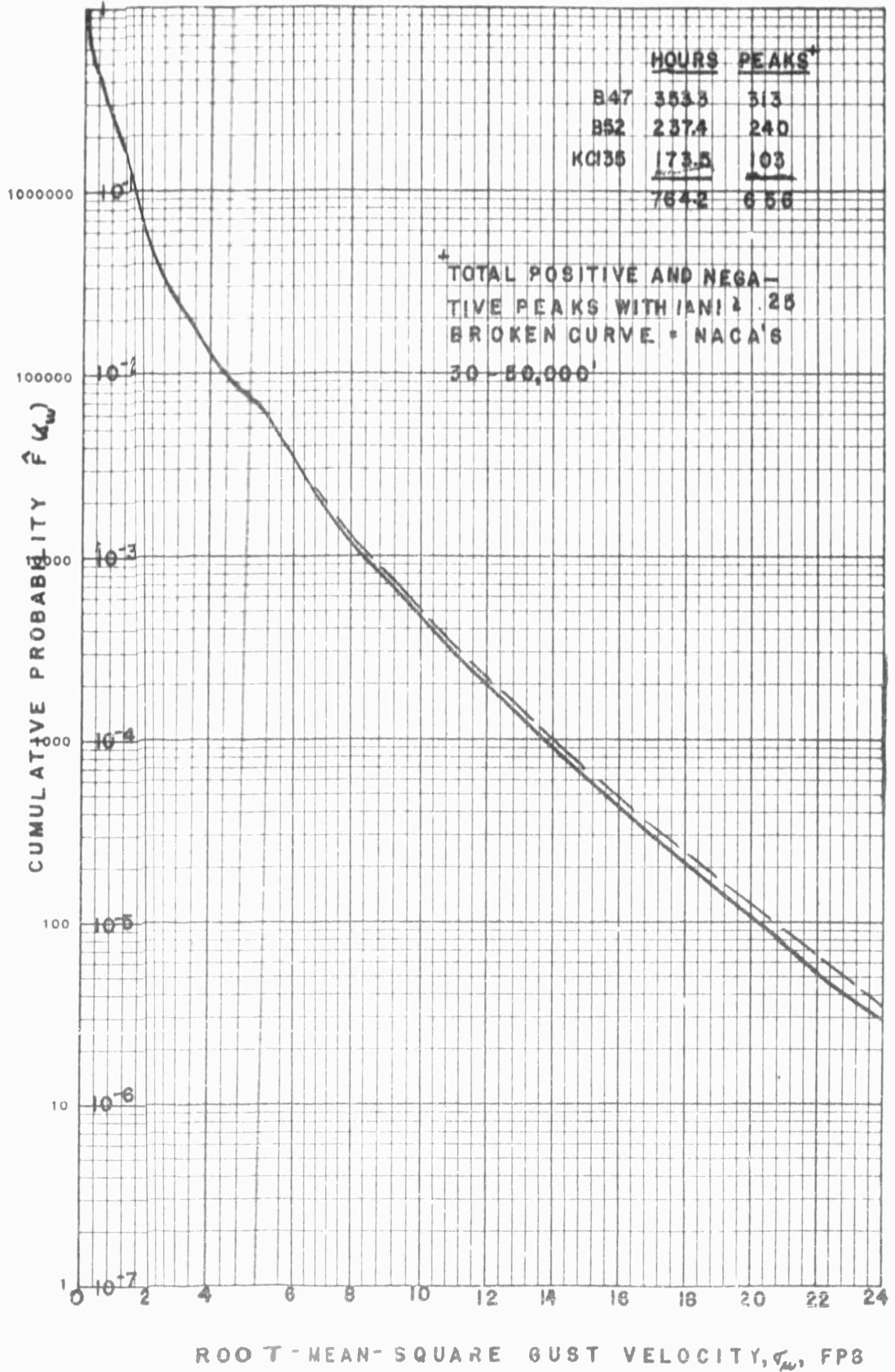
Perhaps the most important points to be noted from these graphs are the relatively large amount of time spent in essentially smooth air at the higher altitudes, and the relatively large amount of time in light to severe turbulence at the lowest altitude bracket (e.g. σ is greater than 2 feet per second 25% of flight time).

COMBINED B47, B52, KCJ35 DATA

ALTITUDE: ABOVE 10,000 FT.

	HOURS	PEAKS[+]
B47	353.3	313
B52	237.4	240
KCI35	173.5	103
	764.2	656

[+]TOTAL POSITIVE AND NEGA-
TIVE PEAKS WITH |ΔN| ≥ .25
BROKEN CURVE = NACA'S
30-50,000'

CUMULATIVE PROBABILITY $\hat{F}(\sigma_w)$

ROOT-MEAN-SQUARE GUST VELOCITY, σ_w, FPS

MODEL

DATE

SUMMARY

Given the variances, it is not yet possible to determine with precision
the effects of turbulence on structures. If the variations are due to
waves with wave lengths of 1 mile, the effects will be quite different
than if the wave lengths were 100 feet. Some characteristic like a
"scale" of turbulence is needed. Here, variance "spectra" will be used
for this purpose. Again, these show what fraction of variance is con-
tributed by various intervals of frequency or wave length. An analogy
is the optical spectrum which shows that fraction of the energy of light
comes from various intervals of frequency or wave length. Statistics
gathered to date were derived from vgh records. The contribution of
atmospheric turbulence to the fatigue of aircraft structures has not
been completely "nailed down" to date - in any rigorous mathematical
sense. However, as shown in the literature and in the practical experi-
ment, good statistical data combined with Power Spectral Density Analysis
has proven quite useful in delineating regions of large turbulence intensity
which do cause considerable damage to aircraft structures.